Strategic Demands for Inventors and the Aggregate Growth

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Increasing share of inventors are employed by frontier firms

Data: PatentsView and Compustat

Notes: 4-digit SIC is used as the definition of industry. Frontier firms in each industry are identified as top revenue firms that jointly account for 50% of total industry revenue. The location of inventors are identified using pre-grant documents data, assuming the first assignee of an application is the employer of inventors.

These Tech Workers Say They Were Hired to Do Nothing

Amid layoffs, former workers in tech are venting about jobs with little to do; 'hoarding us like Pokémon cards'

By Te-Ping Chen Follow

April 7, 2023 5:30 am ET

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Another veteran tech worker, Derrick McMillen, 32, who worked at Facebook and Salesforce before the pandemic, says that during his time at Salesforce, he often felt as though 20% of employees did 80% of the work, while their peers did on-site yoga and took long lunches. Mr. McMillen said he felt that some co-workers there pushed work onto peers, and that anyone pushing back risked being seen as having a bad attitude.

⋮

Keith Rabois, an early PayPal executive and venture capitalist, has accused large tech companies of seeing hiring as a "vanity metric," deliberately hiring talent to keep them from working for other companies. Mr. Rabois made the remarks at a recent event held by the investment banking advisory firm Evercore.

This paper

Innovation deterrence through inventor market?

- Inventor $\rightarrow$ Innovation $\rightarrow$ Improvement in production productivity
- Inventor is the only input of innovation
- Frontier firms can possibly exert market power in input market

Frontier firms' strategic employment decision

- Basic Schumpeterian growth model + Stackelberg competition in inventor market

Does the concentration of inventors accelerate the aggregate growth?

- DRS innovation: $\uparrow$ concentration $\rightarrow \downarrow g$
- Better F's innovation capacity: $\uparrow$ concentration $\rightarrow \uparrow g$

Model

Basic Schumpeterian growth model

Household

- CRRA utility function
- Inelastically supply unskilled and skilled labor

Innovation

- Firms hire inventors and produce innovation

$$x = \theta^{\gamma} h^{\gamma}; \quad \gamma < 1, \quad \theta_F > \theta_L$$

- Step size of innovation is proportional to the average production productivity

$$q_j(t + \Delta t) = q_j(t) + \lambda \bar{q}$$

Output market

- Continuum of industry $[0, 1]$.
- One frontier firm and N_j laggard firms in each industry j.
- Linear production function using unskilled labor as input
- Bertrand competition in output goods market

+ Stackelberg competition in inventor market

Industry-level market segmentation of inventor market

- Strategic decision by frontier firm can be easily modeled.
- Industry-specific skill sets of inventor.
- No inventor mobility of inventors across industries.
- Budget constraint of HH:

$$\dot{A}(t) + C(t) \leq r(t)A(t) + L^u w^u(t) + \int_0^1 L_j^s w_j^s(t) dj.$$

I will assume $L_j^s = L^s$ for all j.

Stackelberg competition in inventor market

Stackelberg competition in inventor market

- In each industry j, the frontier firm acts as the first mover in inventor market.
- Frontier firms optimally choose demand for inventors taking into account
  - Laggard firms' best response
  - Market clearing condition of inventor market

Result of Stackelberg competition

- h_L^*: a laggard firm's optimal demand for inventors
- N: the number of laggard firms.
- Market clearance condition:

$$L^s = h_F^* + N \times h_L^*$$

- h_L^* will be pinned down by parameters (equilibrium result)
- Frontier firm can kill laggards by hiring more inventors

Value functions

Laggard firm

A laggard firm in industry $\hat{q}$ solves the following HJB equation's maximization problem.

$$
\begin{aligned}
rV^L(\hat{q}) =& \xi(\hat{q}) \cdot \left(V^L(\hat{q} + \lambda\bar{\hat{q}}) - V^L(\hat{q}) \right) + \frac{\partial V^L(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} \\
& + \max \left\{ 0, \max_{h_L \geq 0} \left[x_L(h_L) \cdot \left(V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^L(\hat{q}) \right) - w^s(\hat{q}) \cdot (h_L + \phi) \right] \right\}
\end{aligned}
$$

- Effect of other firms' innovation (with intensity $\xi(\hat{q})$)
- Aggregate growth effect: $\uparrow w^u \rightarrow \downarrow \hat{q} = q/w^u \rightarrow \downarrow V^L(\hat{q})$
- Discrete choice over investing in R&D (fixed cost of R&D, ϕ).
- Industry-level skilled wage.

Value functions
Laggard firm

By the definition of relative productivity($\hat{q} = q/w^u$) and the aggregate growth rate ($g = \dot{w^u}/w^u$),

$$\frac{\partial V^L(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u} \frac{\partial w^u}{\partial t} = -g\hat{q}\frac{\partial V^L(\hat{q})}{\partial \hat{q}}.$$

I assume free entry-exit of laggard firms in inventor market.

$$V^L(\hat{q}) = 0, \quad {}^{\forall}\hat{q}.$$

Simplified HJB:

$$\max_{h_L \geq 0} \left[x_L(h_L) \cdot V^F(\hat{q} + \lambda\bar{\hat{q}}) - w^s(\hat{q}) \cdot (h_L + \phi) \right] = 0.$$

- $x_L(h_L) \cdot V^F(\hat{q} + \lambda\bar{\hat{q}})$: expected benefit of R&D
- $w^s(\hat{q}) \cdot (h_L + \phi)$: cost of R&D (wage bill to inventors + fixed cost)

Value functions

Frontier firm - Stackelberg

Let N_j be the number of laggard firms in industry j.

$$
\begin{aligned}
rV^F(\hat{q}) &= \pi(\hat{q}) - g\hat{q}\frac{\partial V^F(\hat{q})}{\partial \hat{q}} + \max\{\mathcal{V}_N, \mathcal{V}_{RD}\}, \\
&\quad \mathcal{V}_N = N_0 x_L(\hat{q}) \cdot (0 - V^F(\hat{q})), \\
&\quad \mathcal{V}_{RD} = \max_{h_F(\hat{q})>0} \Big[- w^s(\hat{q}) \cdot (h_F(\hat{q}) + \phi) \\
&\quad + N_j(h_F(\hat{q})) \cdot x_L(\hat{q}) \cdot (0 - V^F(\hat{q})) + x_F(h_F(\hat{q})) \cdot \Big[V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q}) \Big] \Big]
\end{aligned} \tag{1}
$$

where

$$
N_j(h_F) = \frac{L^s - h_F - \phi}{h_L^* + \phi}, \quad N_0 = \frac{L^s}{h_L^* + \phi}.
$$

Indirectly choosing the number of competitors N_j, frontier firms can reduce the total threat from competitors ($N_j \times x_L$).

Kolmogorov Forward equation

$F(\hat{q})$: Stationary distribution of $\hat{q}$

$$g\hat{q}f(\hat{q}) = \int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q})\ dF(\hat{q}), \tag{2}$$

The above equation gives

$$g = \lambda \int_0^\infty \tau(\hat{q})\ dF(\hat{q}). \tag{3}$$

The average productivity is

$$\bar{\hat{q}} = \int_0^\infty \hat{q}\ dF(\hat{q}).$$

General equilibrium: fixed point $(g, \bar{\hat{q}})$.

Details

Equilibrium

Laggard firm's optimal decision

From laggard firm's HJB equation:

$$h_L^* = \phi, \quad x_L^* = (\theta_L \phi)^\gamma, \quad w^s(\hat{q}) = \gamma \left(\frac{\theta_L}{\phi}\right)^\gamma V^F(\hat{q} + \lambda\bar{\hat{q}}).$$

1. Laggard firms hire more inventors when fixed cost of R&D is high
   - To offset the high fixed cost with high expected benefit of innovation.
   - Zero-value condition should be fulfilled.
2. Skilled wage in industry j is proportional to frontier firm's value in j.
   - The benefit of employing inventor is the increase of expected firm value due to innovation.
   - Innovation value depends on $V^F(\hat{q} + \lambda\bar{\hat{q}})$.

Frontier firm's optimal decision - Competitive

Optimal R&D intensity (demand for inventors)

Similarly, it can be shown that

$$h_F(\hat{q}) = \frac{\theta_F \phi}{\theta_L} \left[\frac{V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})}{V^F(\hat{q} + \lambda\bar{\hat{q}})} \right]^2$$

when frontier firm invests in R&D.

Threshold $\hat{q}$ of R&D

Frontier firm engages in R&D iff

$$\left[\frac{V^F(\hat{q} + \lambda\bar{\hat{q}}) - V^F(\hat{q})}{V^F(\hat{q} + \lambda\bar{\hat{q}})} \right]^2 \geq \frac{\theta_L}{\theta_F}$$

If $V^F(\hat{q})$ is convex, $^{\exists!}\hat{q}_{thr}$ which divides the state space into R&D and non-R&D regions.

- expected benefit of innovation vs. cost of R&D.
- cost of R&D increases when $\hat{q}$ increases.
- Improvement of productivity by innovation $(\lambda\bar{\hat{q}})$ is relatively small when $\hat{q}$ is high.

Frontier firm's optimal decision - Stackelberg

Optimal R&D intensity (demand for inventors)

$$h_F = \frac{\theta_L}{\theta_F}\frac{1}{\phi}, \quad x_F = (\theta_L/\phi)^{\gamma}.$$

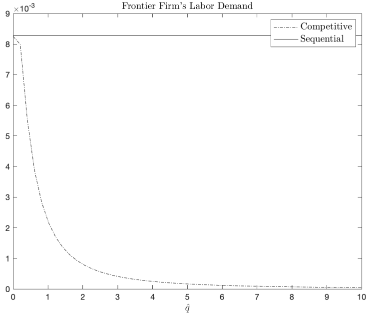
Frontier firm's skilled labor demand depends on

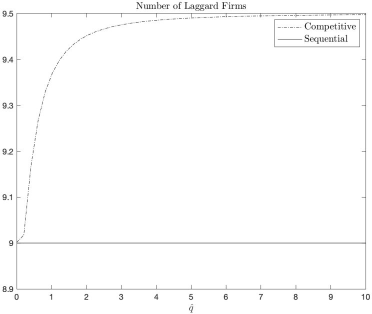
1. Relative innovation capacity θ_F/θ_L.
2. Fixed cost of R&D ϕ.

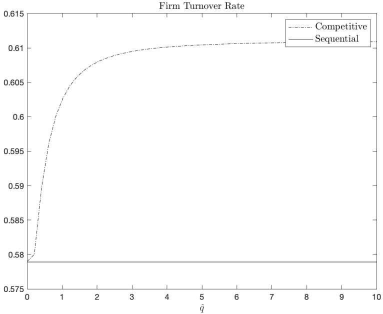
Existence of $\hat{q}_{thr}$?

- In Stackelberg competition, all firms should invest in R&D.
- The additional benefit of reducing the threat from laggard firms makes high-productivity frontier firms invest in R&D.

Computation


Frontier Firm's Labor Demand
×10^-3
Competitive
Sequential
$\hat{q}$


Number of Laggard Firms
Competitive
Sequential
$\hat{q}$


Firm Turnover Rate
Competitive
Sequential
$\hat{q}$

Effect of change in inventor market structure

When input market structure changes from Competitive (C) to Stackelberg (S),

$$g_S - g_C = \underbrace{\lambda \left[\int_0^{\infty} (\tau_S(\hat{q}) - \tau_C(\hat{q})) \cdot dF_C(\hat{q}) \right]}_{\text{misallocation effect}} + \underbrace{\lambda \left[\int_0^{\infty} \tau_S(\hat{q}) \cdot (dF_S(\hat{q}) - dF_C(\hat{q})) \right]}_{\text{distribution effect=0}}.$$

Difference in g can be accounted for by

- Concentration of inventors to frontier
  - $\theta_F > \theta_L$.
  - Decreasing returns to scale of idea production function
- Change in distribution of $\hat{q}$ $\rightarrow$ which is 0 since $\tau_S(\hat{q})$ is constant over $\hat{q}$.
- (Change in L^s)
  - $\uparrow L^s \rightarrow \uparrow \tau(\hat{q})$

Calibration

Calibration

Calibration plan

- I match the competitive model to 1990 U.S. economy
- Switch the input market structure: C $\rightarrow$ S
- Measure the effect of g. Compare S to 2020 U.S. economy

SMM

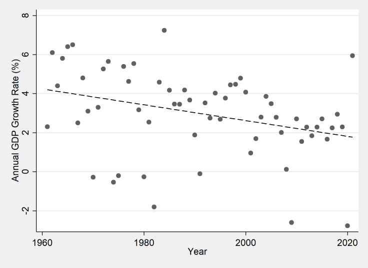
- I calibrate parameters $(\phi, \theta_F, \theta_L)$ of the competitive model to 1990 U.S. economy using SMM.

Table: Parameter Choice

Parameter	Description	Value
ε	Elasticity of substitution	3.00000
L^s	Share of bachelor or higher degree among persons age 25 and over in 1990	0.17018
γ	Innovation elasticity	0.50000
σ	Inverse of IES	2.00000
ρ	Discount rate	0.02000
λ	Innovation step size parameter	0.10000

Target

1. US GDP growth rate in 1990: $g = 0.03431$.



2. Average share of frontier firms' inventor demand in industry:

$$\frac{h_F}{h_F + N \times h_L} = 0.38419.$$

3. Average share of laggard firms' inventor demand in industry:

$$\frac{h_F}{h_F + N \times h_L} = 0.07569.$$

R&D expenditure as proxy of inventor employment

Following Bloom, Jones, Van Reenen, and Webb (2020)

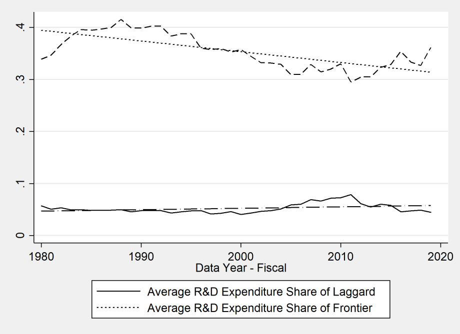


Figure: R&D Expenditure Share of Frontier and Laggard Firm in Industry

Data: Compustat
Notes: 4-digit SIC is used as the definition of industry. Frontier firms in each industry are identified as top revenue firms that jointly account for 50% of total industry revenue. The rest of the firms are identified as laggard firms. The average R&D expenditure is computed using as weight the industry-level total revenue.

Appendix

Household

Household

- Dynamically maximize CRRA utility
- Inelastically supply two types of labors:
  - Unskilled labor ($L^u = 1$): input of output production
  - Skilled labor (L^s): input of innovation/idea

$$U_0 = \int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\sigma} - 1}{1-\sigma} dt$$

$$\text{s.t.} \quad \dot{A}(t) + C(t) \leq r(t)A(t) + w^u(t)L^u + w^s(t)L^s,$$

$$C(t) = \left[\int_0^1 c_j(t)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

Euler equation:

$$g \equiv \frac{\dot{C}}{C} = \frac{r-\rho}{\sigma}.$$

Innovation

Inventor $\rightarrow$ Innovation

- Innovation: technological breakthrough that gives the innovating firm the leading-edge technology
- Poisson arrival intensity of innovation(x) depends on the input of inventors(h).

$$x = \theta^{\gamma} h^{\gamma}; \quad \gamma \in (0, 1).$$

θ: firm-level innovation capacity.

Innovation $\rightarrow$ Productivity growth

- Innovating firm increases the industry production productivity.

$$q_j(t + \Delta t) = q_j(t) + \lambda \bar{q}; \quad \lambda > 0.$$

$q_j(t)$: the best production productivity in industry j at time t.

Output market

Bertrand competition in output market

- Only the frontier firm produces in output market (market for intermediate good j).
- Linear production function using unskilled workers (l).

$$y = ql$$

- The frontier firm maximizes its profit observing inverse demand function for good j.

$$\pi(q) = \frac{1}{\varepsilon - 1}\left(\frac{\varepsilon - 1}{\varepsilon}\right)^{\varepsilon}\left(\frac{q}{w^u}\right)^{\varepsilon - 1}$$

- w^u is the unskilled wage.

$$w^u = \frac{\varepsilon - 1}{\varepsilon} Q, \quad Q = \left(\int q_j^{\varepsilon - 1} dj\right)^{\frac{1}{\varepsilon - 1}}.$$

Balanced growth path

- Normalize all growing variables by w^u; $\hat{q} = q/w^u$

Kolmogorov Forward equation

Computation

(2) is equivalent to

$$g\hat{q}f(\hat{q}) = \int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j)f(\hat{q}_j)d\hat{q}_j.$$

Differentiating both sides with respect to $\hat{q}$ gives

$$g\hat{q}f'(\hat{q}) = (\tau(\hat{q}) - g)\,f(\hat{q}) - \tau(\hat{q}-\lambda\bar{\hat{q}})f(\hat{q}-\lambda\bar{\hat{q}}) \quad (4)$$

This is a delay differential equation with a constant lag $\lambda\bar{\hat{q}}$. This equation is solvable with a history function $f(\hat{q}) = 0$ for all $\hat{q} < \hat{q}_{min}$.

Let $\tilde{f}(\hat{q})$ be the solution of (4). To satisfy the pdf property,

$$\int_{-\infty}^{\infty} f(\hat{q})d\hat{q} = 1,$$

I normalize $\tilde{f}(\hat{q})$ by $\int_{-\infty}^{\infty} \tilde{f}(\hat{q})d\hat{q}$. Since (4) is linear, the normalized function as well as $\tilde{f}(\hat{q})$ satisfies (4).

Back

Proof of the aggregate growth rate

We have the following Kolmogorov forward equation.

$$g\hat{q}f(\hat{q}) = \int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j) \cdot F(d\hat{q}_j)$$

Integrating both sides over $\hat{q}$ gives

$$g\int_0^\infty \hat{q}f(\hat{q})d\hat{q} = \int_0^\infty \left[\int_{\hat{q}-\lambda\bar{\hat{q}}}^{\hat{q}} \tau(\hat{q}_j) \cdot F_t(d\hat{q}_j)\right] d\hat{q} = \lambda\bar{\hat{q}}\int_0^\infty \tau(\hat{q})F(d\hat{q}).$$

From the definition of $\bar{\hat{q}}$,

$$g = \lambda\int_0^\infty \tau(\hat{q})F(d\hat{q}).$$

Back

Computation of value function

Competitive model - Brute force algorithm

1. Starting from an initial guess on $V^F(\hat{q})$.
2. Use the analytic result below about $h_F(\hat{q})$ and evaluate the maximized value of R&D investment.
$$h_F(\hat{q}) = \frac{\theta_F \phi}{\theta_L} \left[\frac{V^F(\hat{q}+\lambda\bar{\hat{q}}) - V^F(\hat{q})}{V^F(\hat{q}+\lambda\bar{\hat{q}})} \right]^2 .$$
3. In each value of $\hat{q}$, compare the evaluated maximum value of R&D investment to the value of not investing. Save this decision in $RD(\hat{q})$ variable.
4. HJB equation is now rephrased as
$$\begin{aligned} rV^F(\hat{q}) =& \pi(\hat{q}) - N(h_F(\hat{q})) \cdot x_L \cdot V^F(\hat{q}) - g\hat{q}\frac{\partial V^F(\hat{q})}{\partial \hat{q}} \\ &+ RD(\hat{q}) \cdot \left[x_F(h_F) \cdot \left[V^F(\hat{q}+\lambda\bar{\hat{q}}) - V^F(\hat{q}) \right] - w^s(\hat{q}) \cdot (h_F + \phi) \right] \end{aligned} \tag{5}$$
5. Find a new fixed point $V^F(\hat{q})$ that satisfies (5) by Least-square algorithm.

Back